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New Approach for the Calculation of Transitional Flows

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In spite of many attempts at modeling natural transition, it has not been possible to predict the streamwise intensities. A procedure is developed that incorporates some results of linear stability theory into one-equation and stress model formulations. The stresses resulting from fluctuations in the transitional region have turbulent, laminar (nonturbulent), and large eddy components. Comparison with Schubauer and Klebanoff's experiments have shown that the nonturbulent and large eddy components have a large influence on the streamwise intensities and little influence on the skin friction. Finally, predictions of the one-equation model were as good as those obtained by the stress model.

Introduction

THE region of transition from laminar to turbulent flow is, in some ways, the least understood and the most important region of fluid flow. In this region, skin friction and heat transfer coefficients change quite rapidly and attain values that exceed those in the fully turbulent region. In general, there are three modes of transition. The first mode is natural transition, which is a result of the Tollmien-Schlichting (TS) mode of instability. Bypass transition, following Morkovin,[1] is caused by large disturbances in the mean flow. The third mode is the separated flow transition that occurs in separated laminar flows. This investigation deals with natural transition.

Models designed to predict transitional flows have been reviewed by Arnal,[2] by Narasimha,[3] and more recently by Scheuerer.[4] A number of the models make use of the intermittency factor γ, which is defined as the fraction of the time that the flow is turbulent at a given point.

Attempts at predicting streamwise intensities were made by, among others, Jiang et al.,[5] who used a one-equation model, and by Arnal and Juillen,[6] who employed a $k - \epsilon$ model. In Ref. 5, empirical modifications were made to the eddy viscosity, the Karman constant, and the length scale in the outer region. Comparisons with measured streamwise intensities pointed to discrepancies in the near wall and outer regions of the boundary layer, particularly for stations central to the transition region. The predictions of Ref. 6 were somewhat poor.

The aim of this investigation is to develop a new approach based on the Navier-Stokes equations to model transitional flows. Two models are employed. The first is a one-equation eddy viscosity model that makes use of the turbulent kinetic energy equation. The second is an abbreviated stress model in which additional equations are solved for the normal stresses. Both of the approaches do not require specification of any special initial conditions or length scales. They do require specification of freestream intensities.

The present investigation addresses the more challenging case of natural transition. In particular, emphasis is placed on modeling the Schubauer and Klebanoff[7] experiment. The start of transition was specified according to the experiment. An expression for intermittency was obtained from Dhawan and Narasimha.[8] The experimental freestream intensity was 0.03 percent.

Formulation of the Problem

Fluctuations in the Transitional Region

If γ represents the fraction of the time that the flow is turbulent, then the mean streamwise velocity U_m is

$$U_m = \gamma U_t + (1 - \gamma) U_\ell \tag{1}$$

where U_t is the mean turbulent velocity and U_ℓ the mean nonturbulent or laminar velocity. Measurements by Kuan and Wang[9] showed that nonturbulent profiles are not Blasius profiles for flows over flat plates. Moreover, turbulent profiles are not the traditional fully developed profiles. As will be discussed subsequently, this is confirmed by calculations. At any given instant, the streamwise fluctuation in the transitional region u'_r is given by

$$u'_r = u - U_m$$

If u is the nonturbulent velocity, then

$$u'_r = U_\ell + u'_\ell - U_m = u'_\ell - \gamma(U_t - U_\ell)$$

and

$$\overline{u'^2_r} = \overline{u'^2_\ell} + \gamma^2 (U_t - U_\ell)^2 \tag{2}$$

Similarly, if u is the turbulent velocity, then

$$\overline{u'^2_r} = \overline{u'^2_t} + (1 - \gamma)^2 (U_t - U_\ell)^2 \tag{3}$$

The expression given in Eq. (3) is obtained γ of the time and the expression in Eq. (2) is obtained $(1 - \gamma)$ of the time. Therefore,

$$\overline{u'^2_r} = \gamma \overline{u'^2_t} + (1 - \gamma) \overline{u'^2_\ell} + \gamma(1 - \gamma)(U_t - U_\ell)^2 \tag{4}$$

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In general, one can write

$$\overline{(u_i'u_j')}_r = \gamma\overline{(u_i'u_j')}_t + (1-\gamma)\overline{(u_i'u_j')}_\ell + \gamma(1-\gamma)\Delta U_i\Delta U_j \tag{5}$$

and

$$k_r = \gamma k_t + (1-\gamma)k_\ell + 2\gamma(1-\gamma)\Delta U_i\Delta U_i \tag{6}$$

when

$$k = \tfrac{1}{2}\overline{u_i'u_i'}, \qquad \Delta U_i = U_{t,i} - U_{\ell,i} \tag{7}$$

Traditionally, terms resulting from nonturbulent fluctuations have never been calculated. This does not rule out use of procedures used to calculate turbulent flows. The terms $\Delta U_i\Delta U_j$ are a result of large eddies. Their calculation requires specification of the turbulent and nonturbulent profiles in the transitional region.

Governing Equations

The equations governing turbulent mean flow are the Reynolds averaged Navier-Stokes equations. When Favre's mass averaging is used, the compressible equations resemble the incompressible equations:

$$\bar\rho_{,t} + (\bar\rho\tilde u_j)_{,j} = 0 \tag{8}$$

$$(\bar\rho\tilde u_i)_{,t} + [\bar\rho\tilde u_i\tilde u_j + \delta_{ij}\bar P - (\bar\tau_{ij} - \overline{\rho u_i''u_j''})]_{,j} = 0 \tag{9}$$

$$(\bar\rho\tilde E)_{,t} + [\bar\rho\tilde u_j\tilde H + \bar q_j - \tilde u_i(\bar\tau_{ij} - \overline{\rho u_i''u_j''}) + \overline{\rho u_j''h''}]_{,j} = 0 \tag{10}$$

where

$$\bar q_j = -\lambda\tilde T_{,j}$$

$$\tilde H = \tilde E + \frac{\bar P}{\bar\rho}$$

$$\bar\tau_{ij} = \mu(\tilde u_{i,j} + \tilde u_{j,i} - \tfrac{2}{3}\delta_{ij}\tilde u_{m,m})$$

$$\bar P = \bar\rho(\gamma - 1)[\tilde E - \tfrac{1}{2}\tilde u_i\tilde u_i] \tag{11}$$

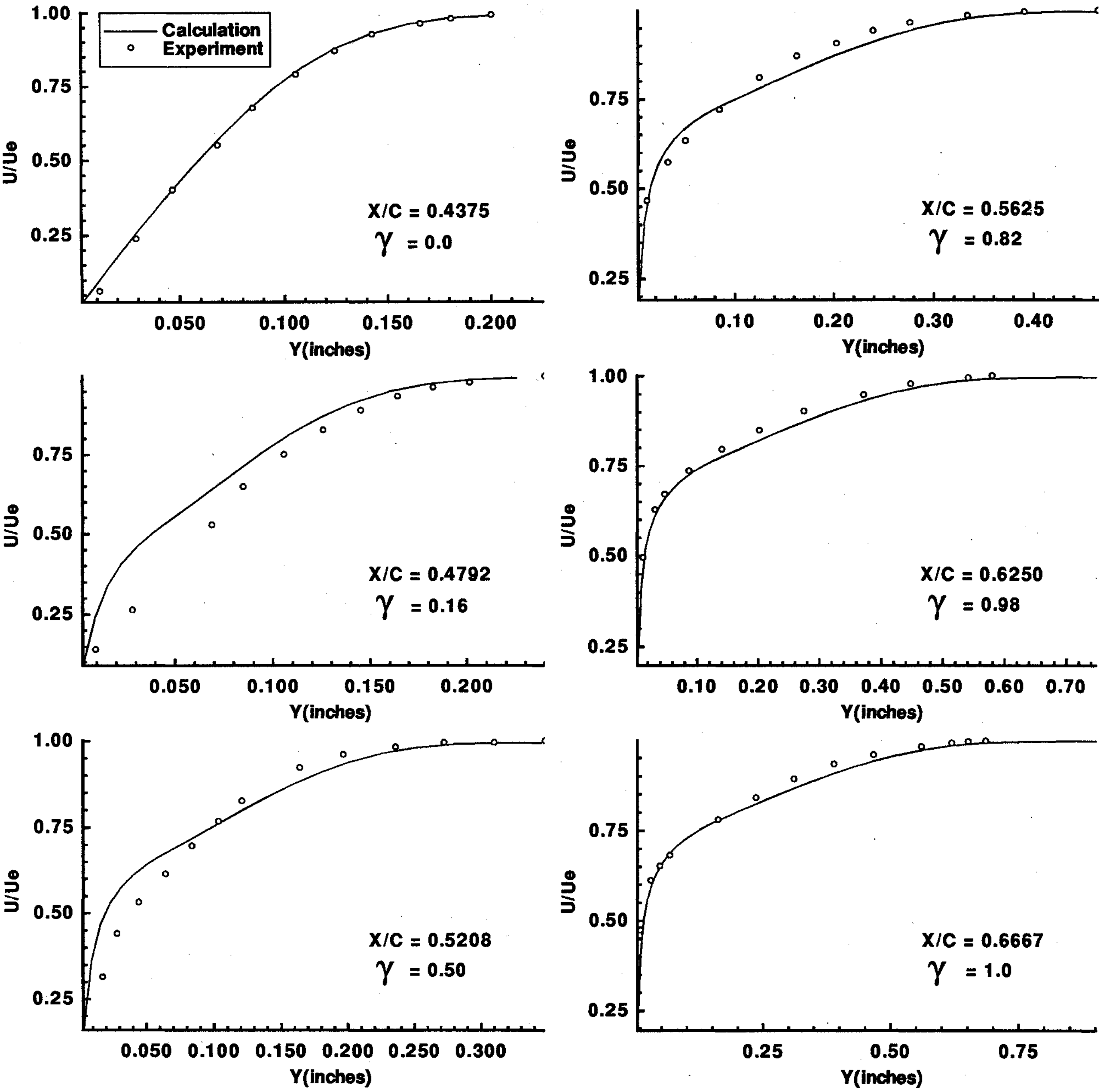

Fig. 1 Typical velocity profiles.

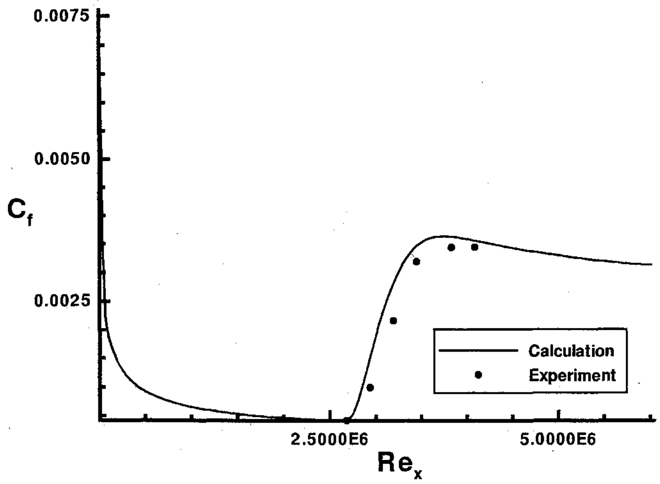

Fig. 2 Typical skin friction.

In the preceding equations, $\bar{\rho}$ is the density, $\tilde{u}_i$ the mean velocity, $\tilde{E}$ and $\tilde{H}$ the total energy and total enthalpy, $\bar{P}$ the pressure, $\tilde{T}$ the temperature, γ the ratio of specific heats, and μ and λ the coefficients of molecular viscosity and thermal conductivity, respectively.

The one-equation model employed follows closely that of Ref. 10. Thus, the turbulent kinetic energy equation is taken as

$$(\bar{\rho}\tilde{k})_{,t} + \left[\bar{\rho}\tilde{u}_j\tilde{k} - \nu(\bar{\rho}\tilde{k})_{,j} - C_k \frac{\tilde{k}}{\bar{\rho}\epsilon}\overline{\rho u_i'' u_j''}\,(\bar{\rho}\tilde{k})_{,i}\right]_{,j}$$

$$= \overline{\rho u_i'' u_k''}\,\tilde{u}_{i,k} - (\bar{\rho}\epsilon) \qquad (12)$$

where

$$\epsilon = \frac{\tilde{k}^{3/2}}{\ell_\epsilon}$$

$$-\overline{\rho u_i'' u_j''} = \mu_t[\tilde{u}_{i,j} + \tilde{u}_{j,i} - \tfrac{2}{3}\delta_{ij}\tilde{u}_{m,m}] - \tfrac{2}{3}\delta_{ij}\bar{\rho}\tilde{k}$$

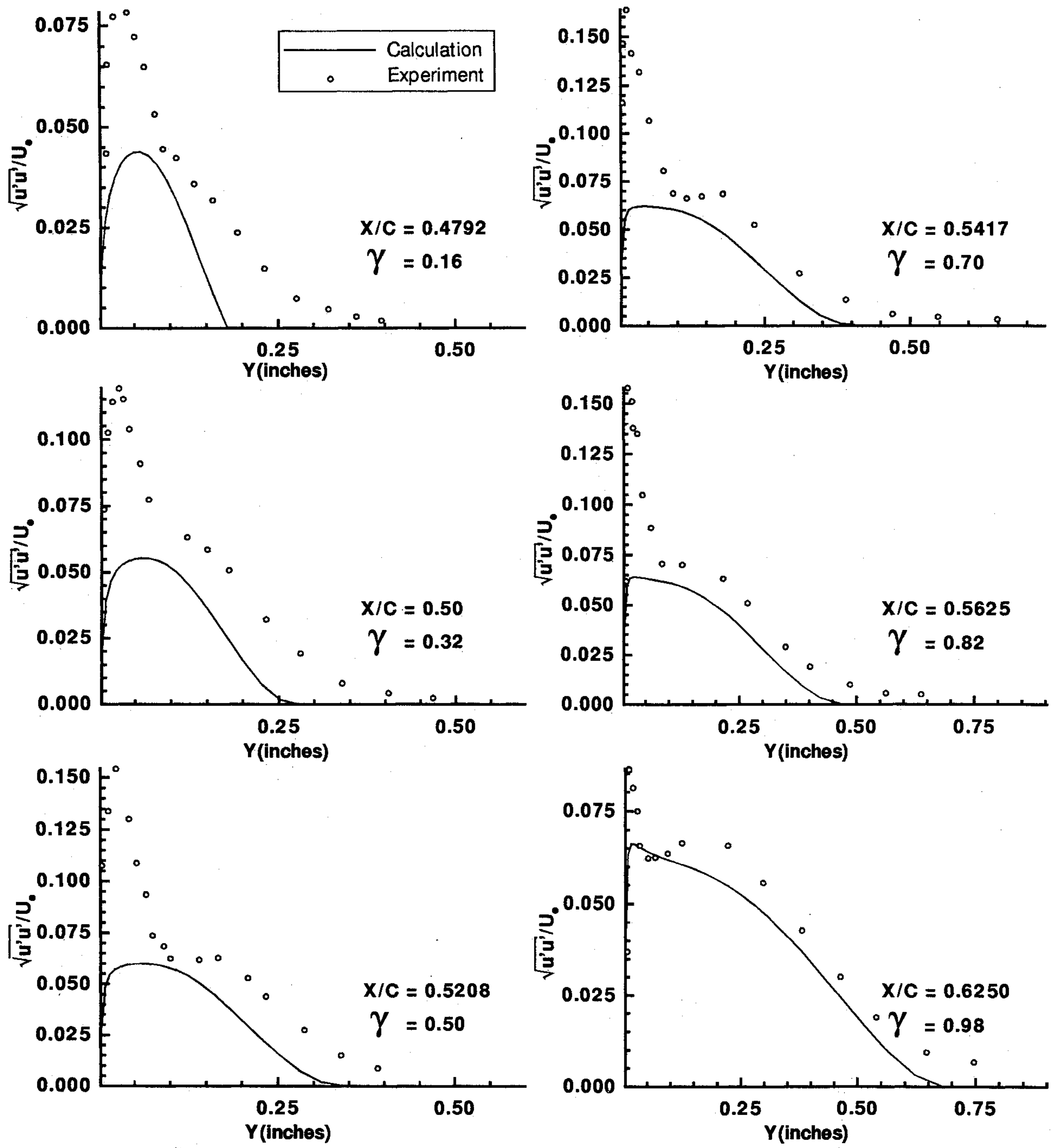

Fig. 3 Streamwise turbulent intensities; contributions of the turbulent normal stresses.

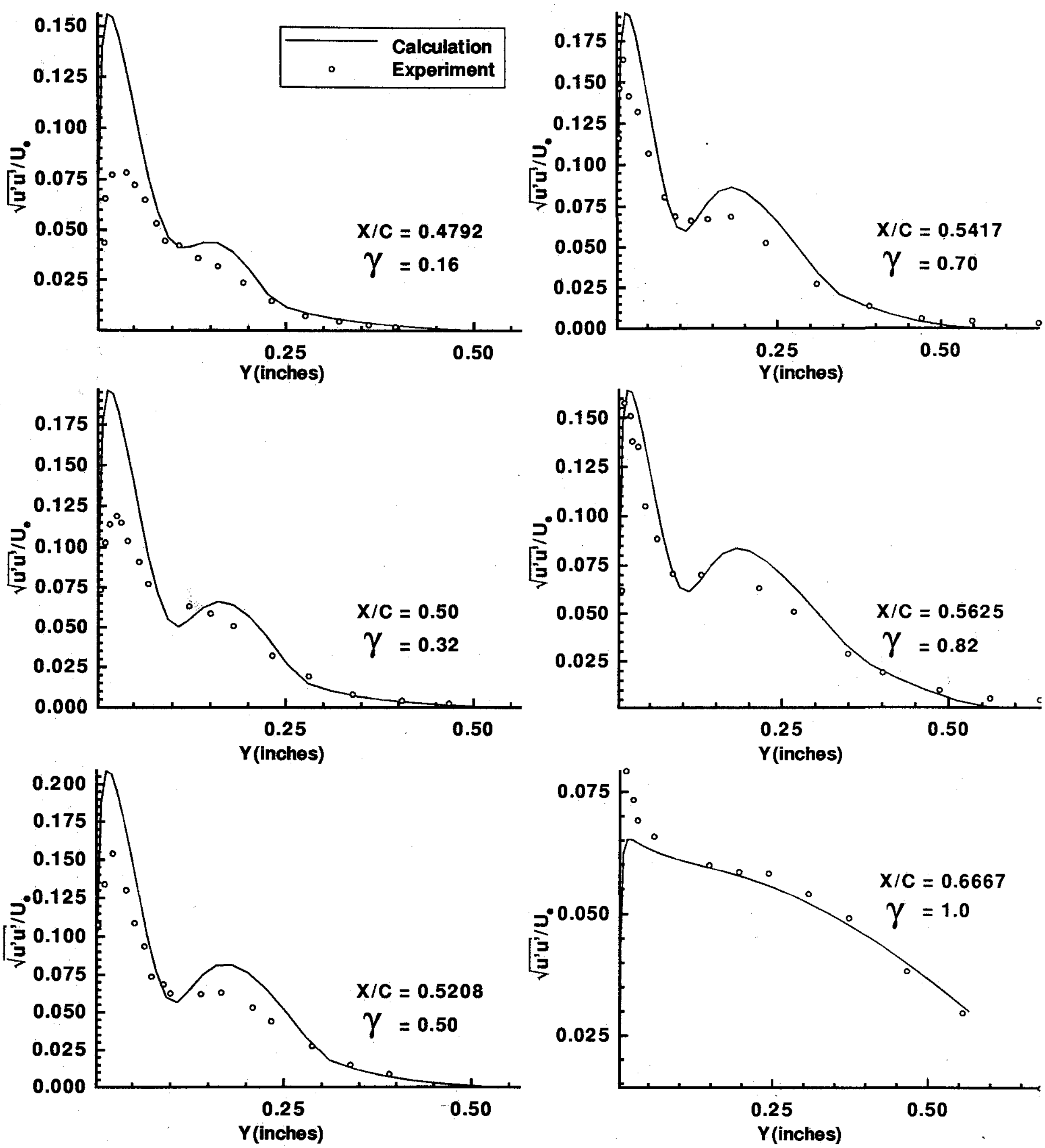

Fig. 4 Streamwise turbulent intensities; influence of large scales [Eq. (18)].

$$\mu_t = C_\mu \bar{\rho} \tilde{k}^{1/2} \ell_{m\mu} \tag{13}$$

ℓ_μ and ℓ_ϵ are the viscosity and dissipation length scales[10] ($C_\mu = 0.09$ and $C_k = 0.25$).

This form of the governing equations can be used for low- and high-speed flows. For the current application, the density is essentially constant and the terms resulting from compressibility corrections can be ignored.

The stress model employed follows closely that of Ref. 11. Because of the complexity of the stress model, these equations will not be reproduced here. Note that in Ref. 11, no ϵ or ω equations were used because the length scale was specified.

Contribution of the TS Waves

Examination of Eq. (5) shows that one needs to model the turbulent contributions, the nonturbulent contributions, and the large scales. The turbulent terms are discussed in the previous section. In general, one can develop a similar formulation for the nonturbulent terms. Traditionally, this has not been done. Because the nonturbulent terms are a result of the presence of the TS waves, an approximate approach that makes use of some of the results of linear stability theory will be adopted here.

The dominant disturbance frequency at breakdown is well predicted by the frequency of the TS waves having the maximum amplification rate. Using the work of Obremski et al.,[12] Walker[13] showed that the locus of maximum amplification rate can be correlated by

$$(\omega\nu/U_e^2) = 3.2Re_{\delta^*}^{3/2} \tag{14}$$

where U_e is the edge velocity, Re_{δ^*} the Reynolds number based on the displacement thickness δ^*, and ω the frequency. It should be noted that Eq. (14) is based on calculations using linear stability theory for the Falkner-Skan profiles and not on experiment. The quantities ω and $\tilde{k}$ determine a length scale ℓ_{ts} given by

$$\ell_{ts} = a\frac{\tilde{k}^{1/2}}{\omega} \tag{15}$$

where a is a constant to be determined later. Thus, one way to account for the presence of the TS waves and the resulting nonturbulent fluctuations is to solve an equation for $\tilde{k}_\ell$ similar to that of $\tilde{k}_t$ but with ℓ_μ replaced by ℓ_{ts}. This was not done here. Instead it was assumed that both turbulent and nonturbulent contributions can be obtained by replacing ℓ_μ in Eq. (13) by

$$\ell_\mu = (1-\gamma)\ell_{ts} + \gamma\ell_\mu^t \tag{16}$$

where ℓ_μ^t is the turbulent viscosity length scale. This approach gives the transitional shear stress directly.

Before transition, where $\gamma = 0$, $\ell_\mu = \ell_{ts}$. Therefore, an estimate of a can be obtained from a calculation of streamwise intensities before transition and comparison with the data of Ref. 7. Using this procedure, a value of 0.07 is obtained.

Contribution of the Large Scales

In order to determine the contribution of the larger scales, i.e., terms $\Delta U_i \Delta U_j$ in Eq. (5), a number of procedures were considered. The first assumes that the nonturbulent profiles are Blasius profiles and the turbulent profiles are the fully developed turbulent profiles. As will be seen from the results section, this assumption is not satisfactory.

The second procedure considered made use of the boundary layer code developed by Harris.[14] This code was exercised in the following manner. Starting from an assumed transition point x_t and an assumed γ distribution, calculations were carried out downstream of x_t by assuming that the flow is either laminar or turbulent. The choice is determined by whether γ is larger than a random number R_f. If $\gamma \geq R_f$, the turbulent version of the code is used to calculate the flow

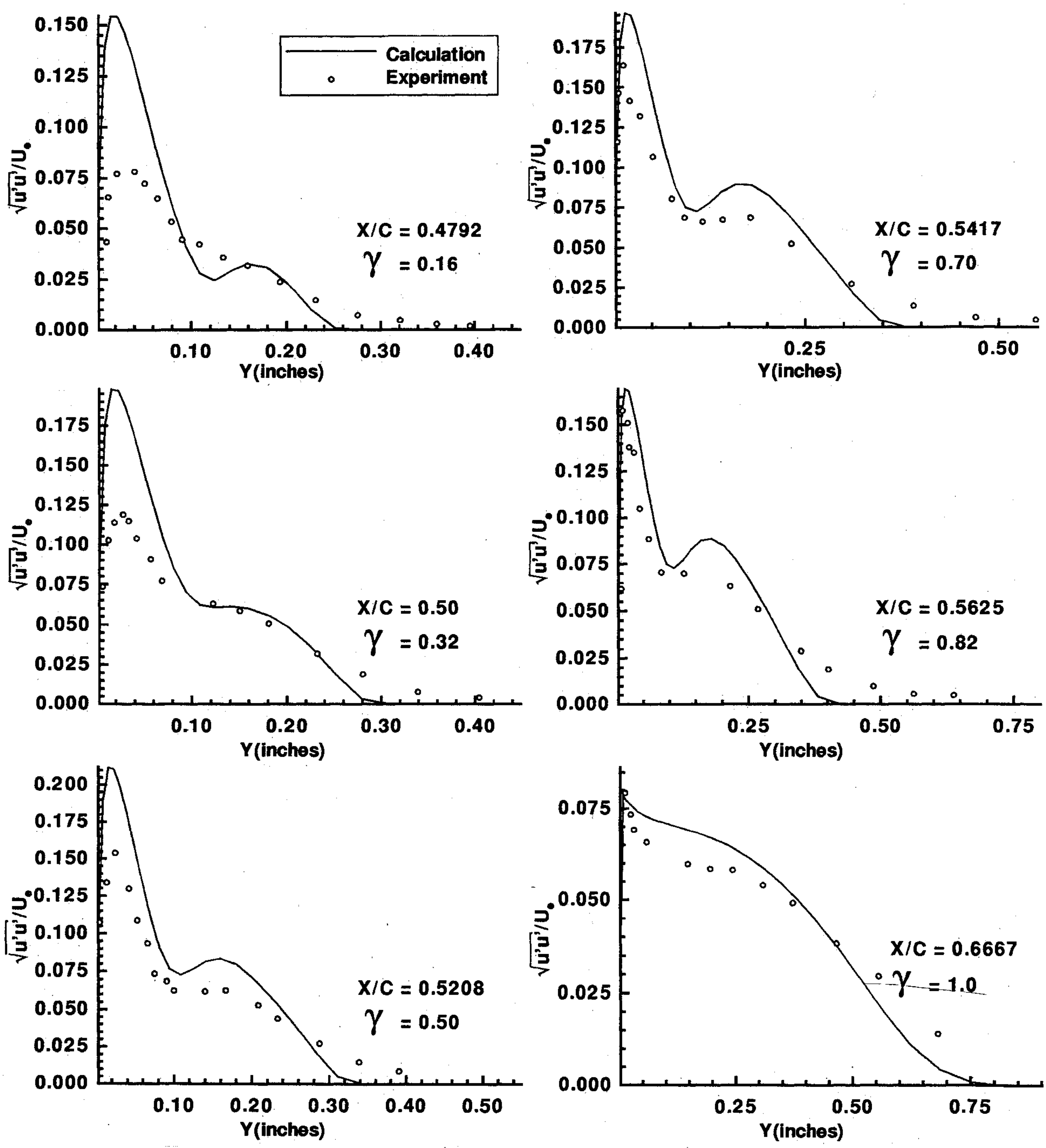

Fig. 5 Streamwise turbulent intensities—stress model; influence of large scales [Eq. (18)].

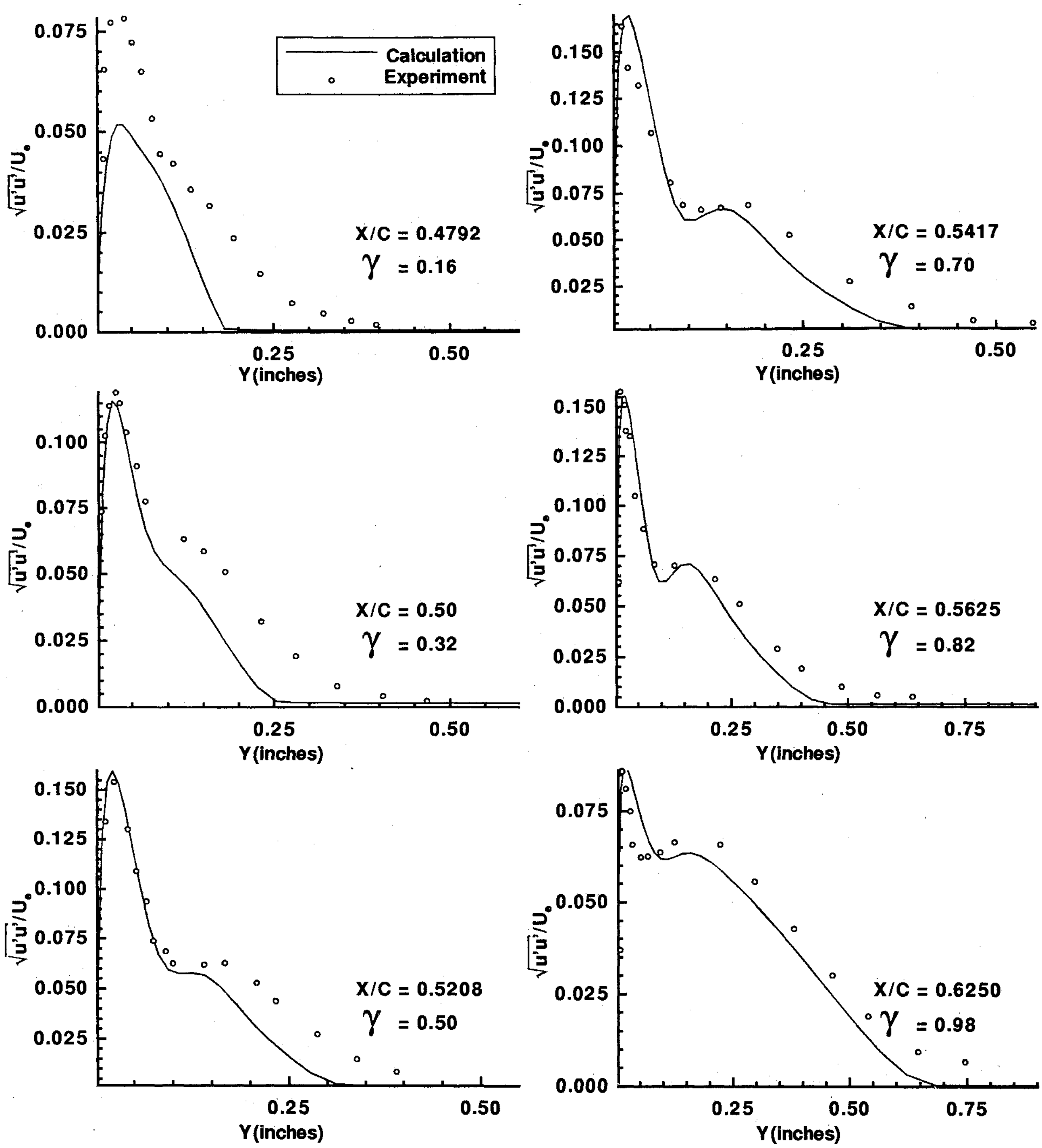

Fig. 6 Streamwise turbulent intensities; influence of large scales [Eq. (17)].

there. If, on the other hand, $\gamma < R_f$, then the flow is treated as laminar. Because of the marching nature of boundary layer codes, calculated laminar profiles quickly become similar to the turbulent profiles. As a result, ΔU_i calculated from this procedure were not satisfactory, and results of this procedure are not included.

The third procedure assumes ΔU_i as

$$\Delta U_i = U_i - (U_i)_{\gamma=0} \tag{17}$$

The rationale for this choice stems from the fact that $(U_i)_{\gamma=0}$ is representative of the nonturbulent profiles and U_i are not much different from turbulent profiles. In the absence of a conditional sampling procedure, it is difficult to calculate actual turbulent and nonturbulent profiles in the transitional region.

Results and Discussion

An upwind procedure similar to that developed in Ref. 11 is used for both one-equation and stress models. Because of the low Mach numbers considered, grid sequencing was used to accelerate convergence. All comparisons are made with the experiments of Ref. 7. Unless otherwise noted, calculations are based on the one-equation model.

Velocity profiles and skin friction calculations were somewhat insensitive to the contribution resulting from TS waves and the large scales. Moreover, they were insensitive to the type model employed, i.e., one-equation vs stress model. Figures 1 and 2 show a comparison of a representative calculation with experiment. Figure 1 compares velocities, whereas Fig. 2 compares skin friction. As the figures show, both predictions are in good agreement with experiments. Thus, if the objective

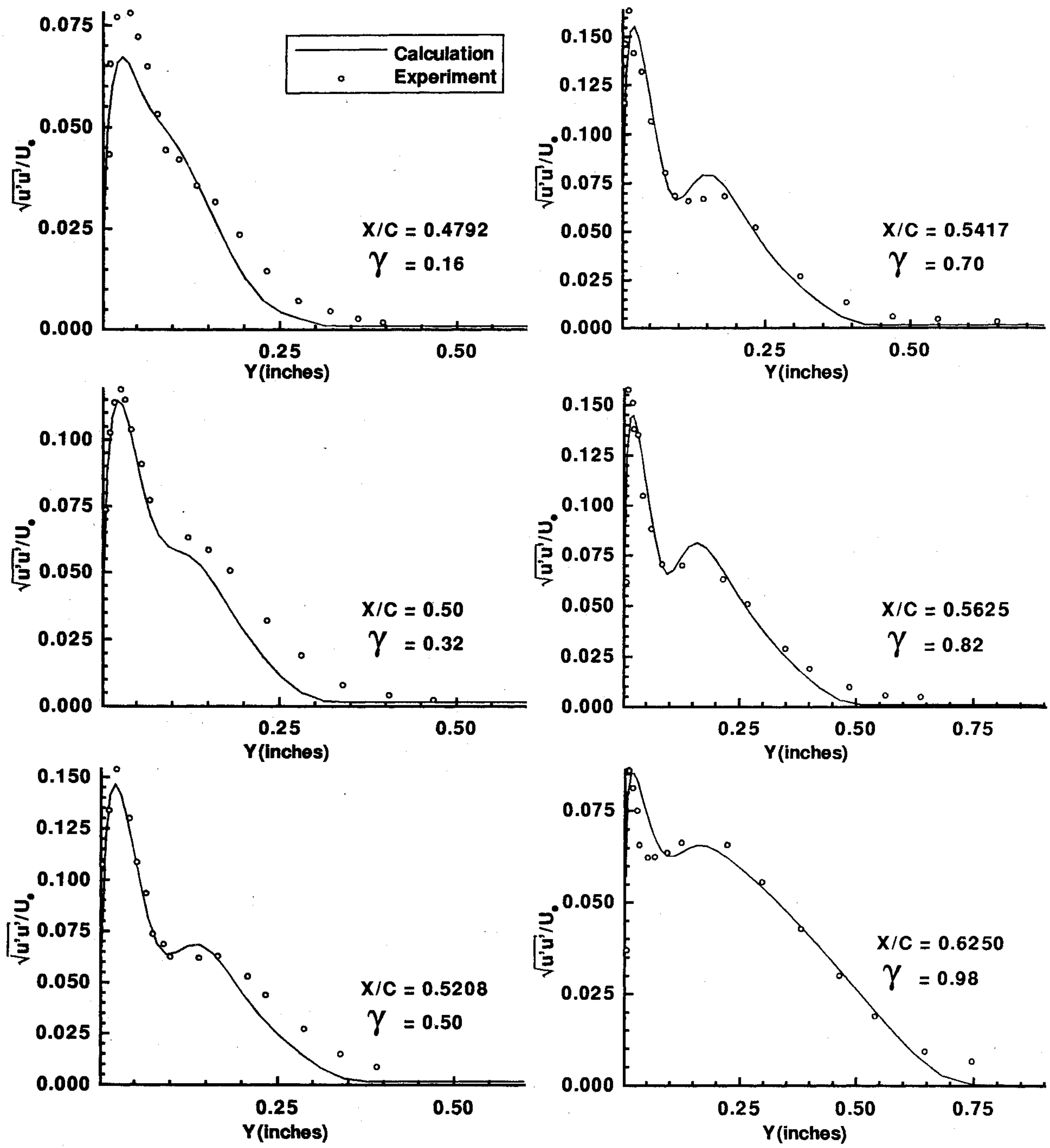

Fig. 7 Streamwise turbulent intensities; influence of large scales [Eq. (17)] and TS correction.

is the calculation of skin friction and velocity profiles, one can approximate the transitional shear stress term by the first term in Eq. (5).

Initially, solutions were obtained using only the first term of Eq. (5). Figure 3 compares the resulting streamwise intensities with experiments. As the figure shows, large discrepancies exist, especially in the near wall region. This shows that approximating transitional normal stresses by their turbulent component is not a good approximation.

The remaining discussion will concentrate on the influence of the TS waves and large scales on intensity. Assumptions regarding large scales are considered first. Figures 4 and 5 were obtained using the one-equation model and the stress model, respectively, for the case where ΔU_i is given by

$$\Delta U_i = U_{i,t} - U_{i,\ell} \tag{18}$$

where U_t and U_ℓ assume fully developed turbulent profiles and Blasius profiles, respectively. In this calculation, the contribution of the TS correction was neglected. As the figures show, this particular choice of ΔU_i overpredicts streamwise intensities. Thus, such an assumption is unsatisfactory. In addition, nothing is gained by using a stress model in place of a one-equation model. Because of this, subsequent comparisons will be restricted to results of the one-equation model.

The next set of calculations assumes that the large scales are modeled by Eq. (17). Figure 6 shows a calculation without allowing for TS waves, whereas Fig. 7 allows for the TS waves. The two figures show that allowing for the presence of the TS waves improves predictions in the early stage of transition. Moreover, they show that the presence of the spikes in the streamwise intensities are a direct result of the large scale eddies. Further, the choice of Eq. (17) to represent the contributions of the large scales appears to be a satisfactory approximation.

Concluding Remarks

This investigation shows that large scales and TS waves have little influence on the transitional skin friction but have a profound influence on the intensities. The approach used to account for both of these effects is somewhat approximate. More elaborate models would be needed to improve the present predictions. It is expected that both of these mechanisms will not be very important when the bypass mechanism is in effect, i.e., at high freestream turbulent intensities.

Acknowledgments

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